

# A Study of The Effect of Design Variable Scaling on The Performance of an Aerodynamic Shape Optimization Algorithm

Howard P. Buckley \*

*Institute for Aerospace Studies, University of Toronto  
4925 Dufferin St., Toronto, Ontario, M3H 5T6, Canada*

**Try several different design variable scaling approaches for several different optimization test cases to see their effect on optimization performance. Also look at the effect of normalizing the objective function by its initial value. Try 2D and 3D optimization cases.**

## I. Introduction

A study of design variable scaling approaches has previously been performed by Zingg et al.<sup>?</sup> Their study compared performance of several 2D aerodynamic shape optimization test cases using a combination of design variable and Hessian scaling approaches. The optimizations were performed using the BFGS optimization algorithm within Optima2D. Since then, the SNOPT optimization algorithm for constrained optimization problems has become the preferred method for optimizations with Optima2D. This study revisits the design variable scaling question to determine the best scaling approach for optimization when using SNOPT.

## II. Description of Design Variable Scaling Approaches

### A. Scaling 0

No scaling applied to any design variables:

- SC\_METHOD = 3

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\*Research Engineer, howard@oddjob.utias.utoronto.ca

- $\text{GDV\_SCALE} = 1.0$
- $\text{AOADV\_SCALE} = 1.0$

## **B. Scaling 1**

Ad-hoc scaling factors applied to geometric and AoA design variables:

- $\text{SC\_METHOD} = 3$
- $\text{GDV\_SCALE} = 0.1$
- $\text{AOADV\_SCALE} = 1.0$

## **C. Scaling 2**

Ad-hoc scaling factors applied to geometric and AoA design variables:

- $\text{SC\_METHOD} = 3$
- $\text{GDV\_SCALE} = 0.01$
- $\text{AOADV\_SCALE} = 1.0$

## **D. Scaling 3**

Ad-hoc scaling factors applied to geometric and AoA design variables:

- $\text{SC\_METHOD} = 3$
- $\text{GDV\_SCALE} = 0.001$
- $\text{AOADV\_SCALE} = 1.0$

## **E. Scaling 4**

Ad-hoc scaling factors applied to geometric and AoA design variables:

- $\text{SC\_METHOD} = 3$
- $\text{GDV\_SCALE} = 10.0$
- $\text{AOADV\_SCALE} = 1.0$

## F. Scaling 5

Ad-hoc scaling factors applied to geometric and AoA design variables:

- $SC\_METHOD = 3$
- $GDV\_SCALE = 100.0$
- $AOADV\_SCALE = 1.0$

## G. Scaling 6

Scale all design variables so that they are of order 1e0:

- $SC\_METHOD = 3$
- $GDV\_SCALE = 0.0$  (Setting this parameter to zero scales all geometric design variables by the average of their initial absolute values)
- $AOADV\_SCALE = 14.0$  (For multipoint problems with low-speed, high-lift off-design constraints, AoA is a design variable. For example see O2D test suite case 024. The AoA that satisfies the lift constraint is 14 degrees)

## H. Scaling 7

Scale each geometric design variable by its initial value. AoA design variables remain unscaled:

- $SC\_METHOD = 2$
- $GDV\_SCALE = N/A$
- $AOADV\_SCALE = N/A$

# III. Description of Test Cases

## A. 2D Cases from the Optima2D Test Suite

- 014: single pt, 10 dvs, range thickness + 2 TE thickness constraints
- 016: same as 014, but with cold starts
- 017: same as 014, but with 28 design variables
- 024: 18 pt multipt opt, area constraint + 2 TE thickness constraints, initial AoA values for high-lift points = 14.5 degrees

- 024-2: same as 024, but with initial AoA values for high-lift points = 5.0 degrees
- 025: weighted integral case with 27 on-design points plus 10 off-design points

## B. 3D Cases

- BWB1: One of Nimeeshas 3D BWB optimization cases

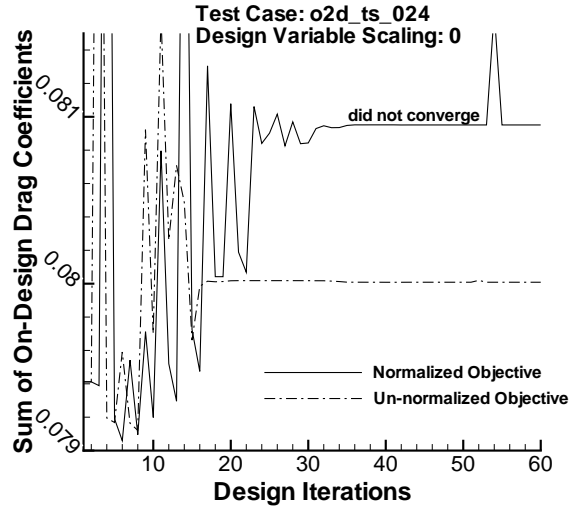
# IV. Results

## A. Test Case 024 from Optima2D Test Suite

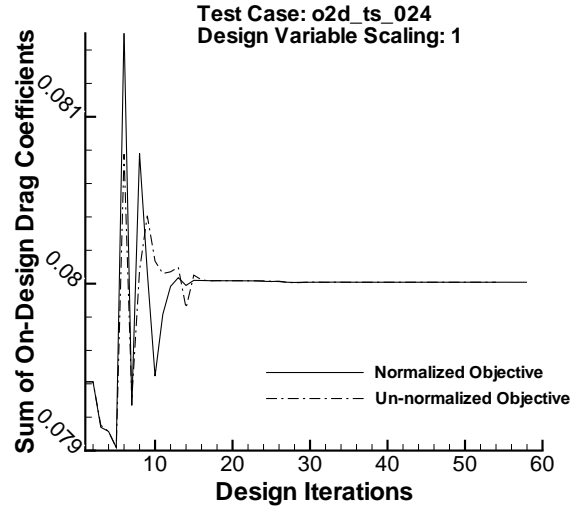
Figures 1 and 2 show a comparison of optimization performance for this test case. Each subfigure compares the performance of an optimization executed using an objective function normalized by its initial value versus performance using an un-normalized objective function for a given design variable scaling approach. Design Variable Scaling 2 using an un-normalized objective function converges in the fewest number of design iterations. The objective function is converged to within 99.9% of its final value and all constraints are satisfied after 12 design iterations. It is worth noting that for all design variable scaling approaches, an un-normalized objective function performs at least as well as with a normalized objective function and in some cases has far superior performance.

## B. Test Case 014 from Optima2D Test Suite

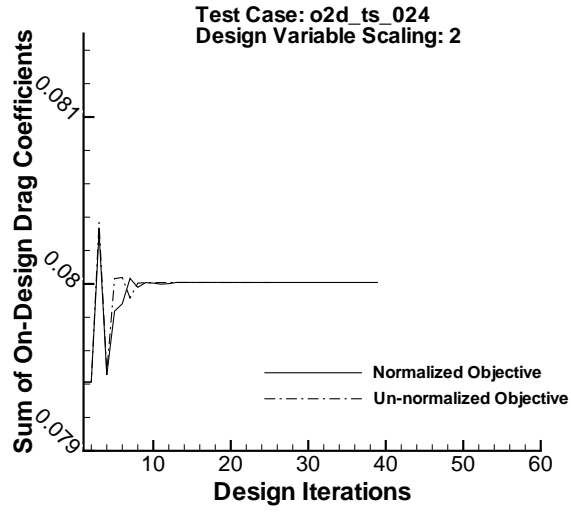
Figures 3 and 4 show a comparison of optimization performance for this test case. Each subfigure compares the performance of an optimization executed using an objective function normalized by its initial value versus performance using an un-normalized objective function for a given design variable scaling approach. Optima2D test suite case 014 is a single point optimization with a lift-constrained drag-minimization objective function. The airfoil geometry is parameterized with 15 bspline control points. 10 of these are used as design variables. There are no angle of attack design variables. For this test case, only the geometric design variable scaling, GDV\_SCALE, is applicable. For every design variable scaling approach, the path to the optimal solution is nearly identical when comparing results using a normalized objective function versus an un-normalized objective function. For all but one of the design variable scaling approaches, the objective function is converged to within 99.9% of its final value and all constraints are satisfied within 15 design iterations. The notable exception is design variable scaling approach 7 which does not achieve a converged result until after 28 design iterations. Design variable scaling approach 7 differs from the rest as it is the only approach that does not scale all design variables by a common factor, rather it scales each design variable by its initial value.



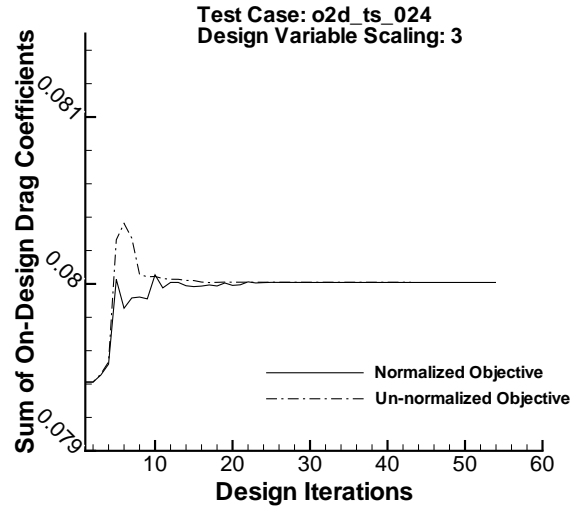
(a) Design Variable Scaling 0



(b) Design Variable Scaling 1



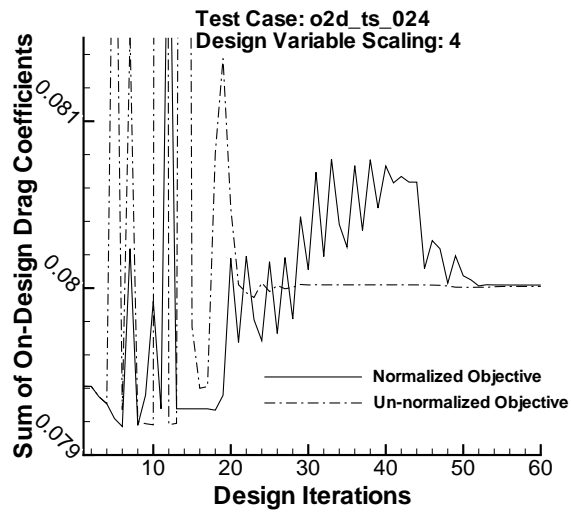
(c) Design Variable Scaling 2



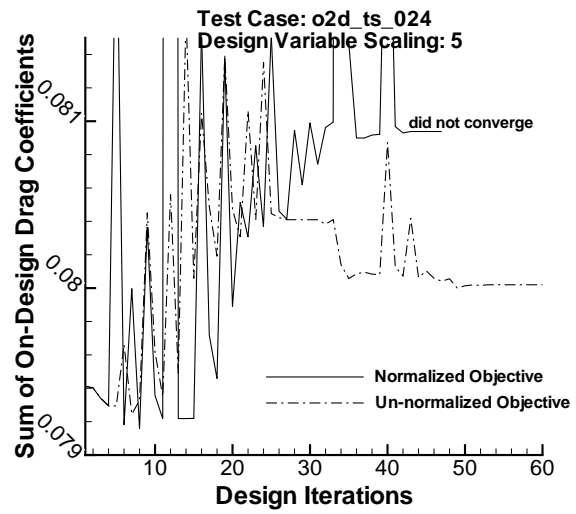
(d) Design Variable Scaling 3

Figure 1. Comparison of design variable scalings 0 - 3 for test case 024 from the Optima2D Test Suite)

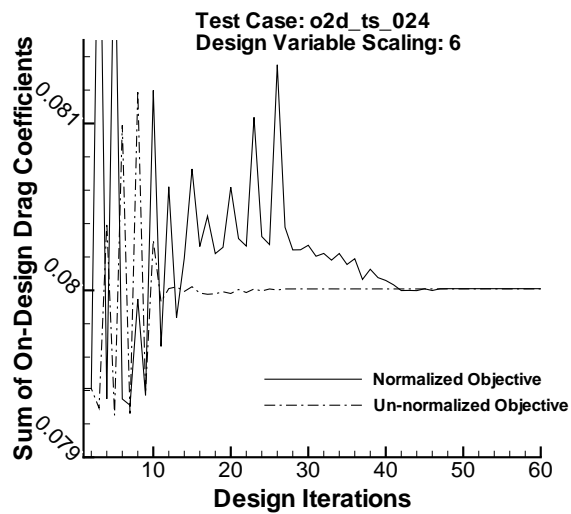
## V. Conclusions



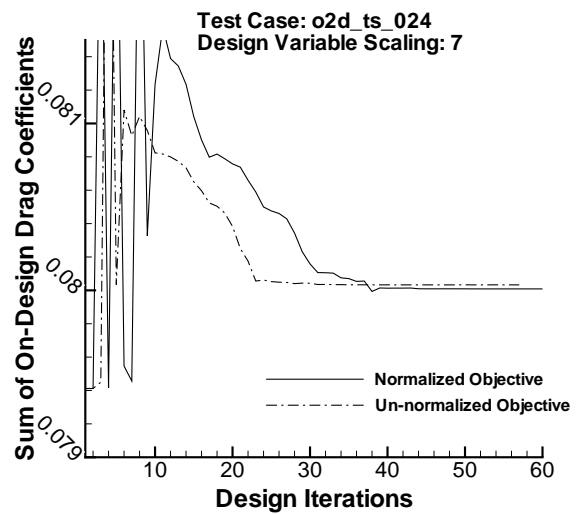
(a) Design Variable Scaling 4



(b) Design Variable Scaling 5

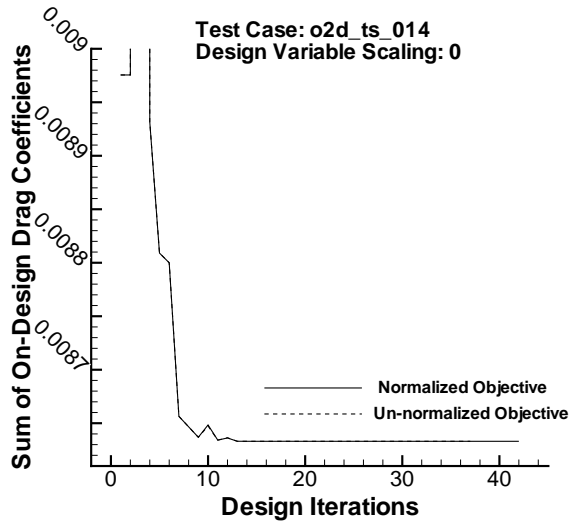


(c) Design Variable Scaling 6

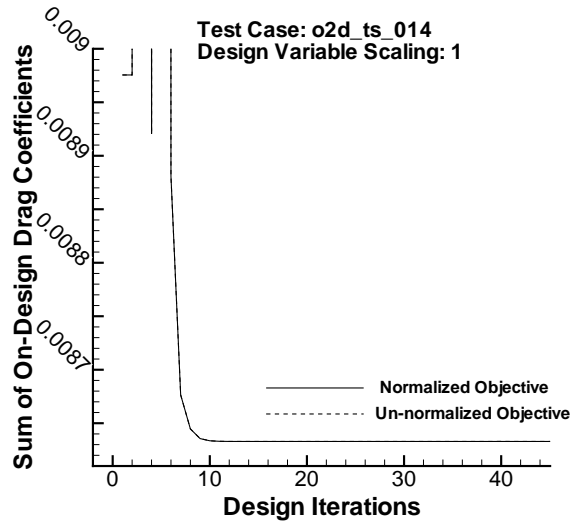


(d) Design Variable Scaling 7

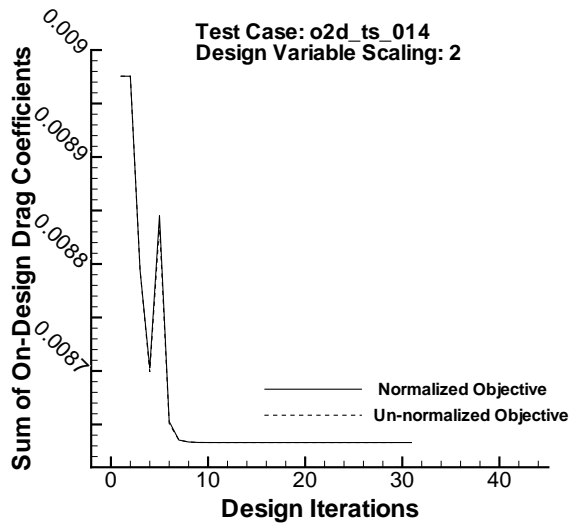
Figure 2. Comparison of design variable scalings 4 - 7 for test case 024 from the Optima2D Test Suite)



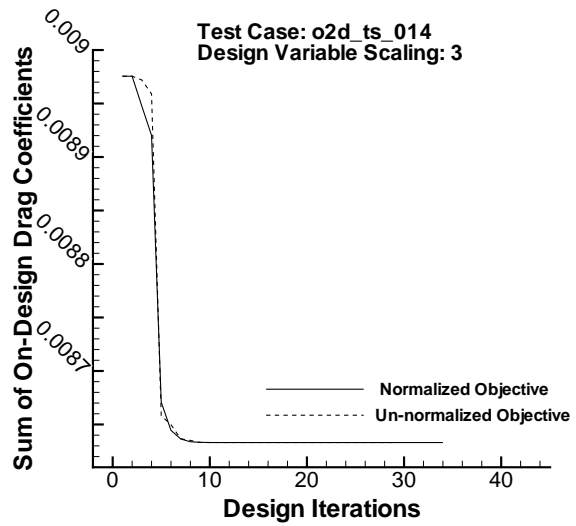
(a) Design Variable Scaling 0



(b) Design Variable Scaling 1

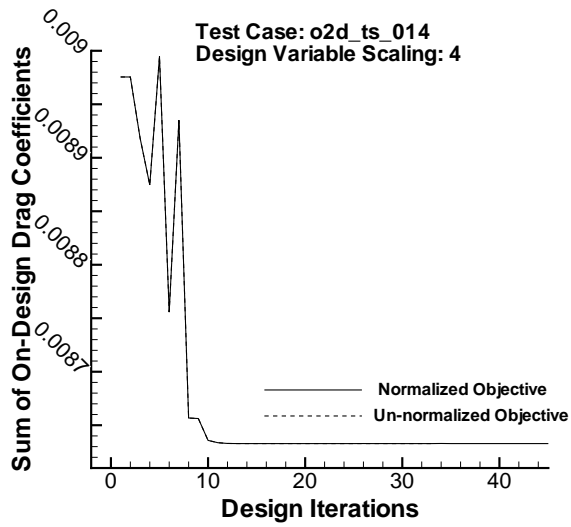


(c) Design Variable Scaling 2

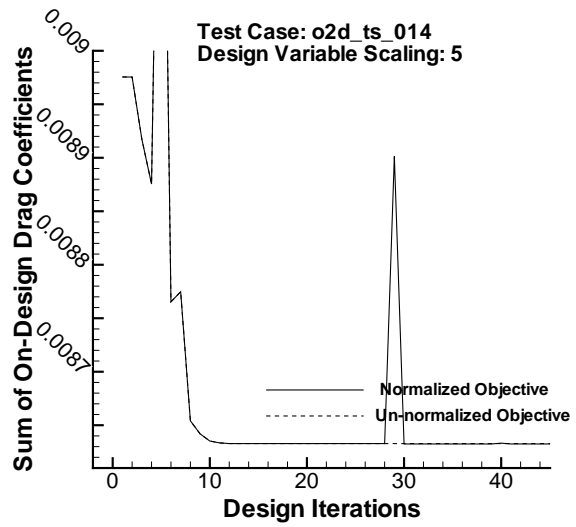


(d) Design Variable Scaling 3

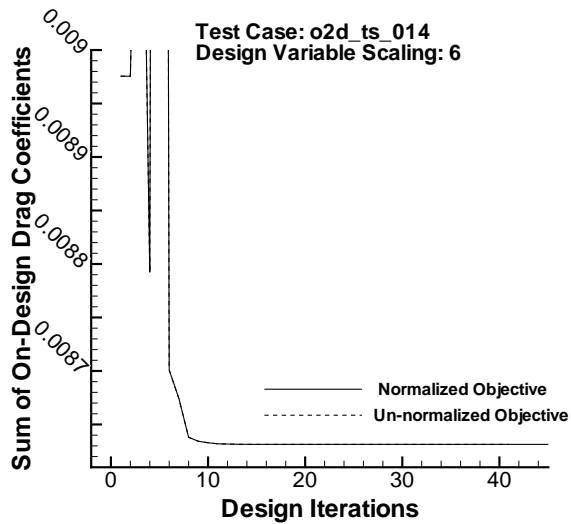
Figure 3. Comparison of design variable scalings 0 - 3 for test case 014 from the Optima2D Test Suite)



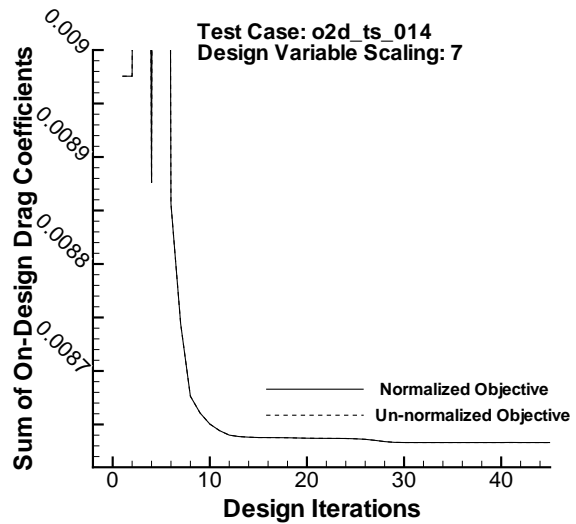
(a) Design Variable Scaling 4



(b) Design Variable Scaling 5



(c) Design Variable Scaling 6



(d) Design Variable Scaling 7

Figure 4. Comparison of design variable scalings 4 - 7 for test case 014 from the Optima2D Test Suite)